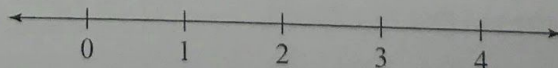


Learning Exercises for Section 6.1

1. In how many ways can you "cut" a square region into two equal pieces?
2. Using a rectangular or circular region as the whole, make sketches to show each quantity:
a. $\frac{2}{3}$ b. $\frac{7}{4}$ c. $2\frac{3}{8}$ d. $\frac{0}{4}$ e. $\frac{3}{1}$
3. a. Mark this representation of 4 feet of a licorice whip to show $\frac{3}{4}$ of the licorice whip.



- b. Mark $\frac{3}{4}$ on the number line.



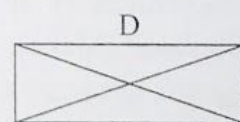
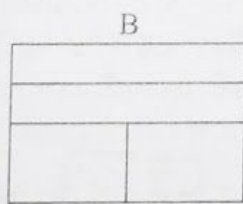
- c. Explain the difference in parts (a) and (b) by referring to the unit.
4. It is important to keep in mind what the unit or whole is for a particular fraction.
Make a drawing for this situation: John ate $\frac{1}{4}$ of a cake. Maria ate $\frac{1}{4}$ of the remaining

cake. What was the unit or whole in John's case? In Maria's case? How much of a complete cake was there in each case?

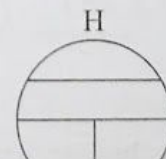
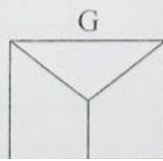
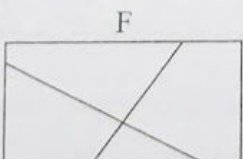
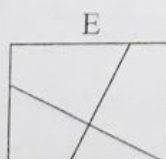
5. On a number line, show $\frac{3}{2}$ using the following views:
 - a. the part-whole view
 - b. the division view
6. **Pattern blocks** Using pattern blocks, get the yellow hexagon, the red trapezoid, the blue rhombus, and the green triangle. (Cut-outs of the pattern blocks are included in the "Masters" available online at www.macmillanlearning.com/reconceptmath3e. Depending on the colors used, they may or may not be the colors indicated here.)
 - a. Let the hexagon = 1. Give the value for each of the other three pieces.
(Note: The = sign means "represent.")
 - b. Let the trapezoid = 1. Give the value for each of the other three pieces.
 - c. Let a pile of two hexagons = 1. Give the values for the hexagon and each of the other three pieces.
7. **Pattern blocks** In how many different ways can you cover the hexagon? Write an addition equation for each method.
8. Using drawings of circular regions, justify that each expression is true:
 - a. $\frac{5}{8} > \frac{1}{2}$
 - b. $\frac{5}{8} < \frac{3}{4}$
 - c. $2\frac{5}{6} > \frac{5}{2}$
9. Use drawings of rectangular regions to find the larger fraction of each pair:
 - a. $\frac{2}{3}$ and $\frac{3}{4}$
 - b. $\frac{1}{2}$ and $\frac{4}{7}$
 - c. $\frac{1}{8}$ and $\frac{1}{9}$

Using the part-whole meaning of a fraction, find the larger fraction of each pair. Write down your explanation.

 - d. $\frac{12}{17}$ and $\frac{12}{19}$
 - e. $\frac{10}{9}$ and $\frac{11}{9}$
 - f. $\frac{62}{101}$ and $\frac{62}{121}$
10. You are in charge of the Student Education Association cake sale. As you leave to buy some paper plates, you ask someone you do not know to cut all the cakes. When you return and look at the cakes (see the diagrams below), you gasp! The plan had been to charge \$5 for a whole cake, and you had thought that each piece could be priced by dividing \$5 by the number of pieces, if necessary.
 - a. For each cake, is it fair to charge the same amount for each piece? (Do not worry about how the icing is spread or whether a piece close to an edge might be a little smaller in height.)

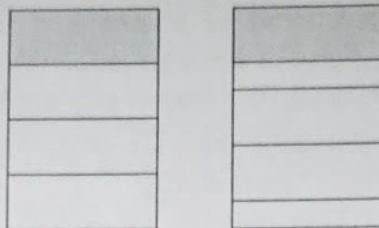


Cake C has cuts evenly spaced.

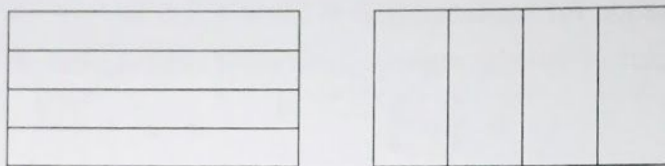


- b. Cut your own cake *creatively* but so that every piece is worth the same amount.
 c. Which of your fair/not fair decisions in diagrams A–L would change if the price of a whole cake changed from \$5 to some other amount?
 d. What does part (a) have to do with fractions?

11. Can you say what fraction of the second rectangular region is shaded? Explain.



12. a. If you were offered a piece from one of the following cakes, which cake would give you the larger piece?¹

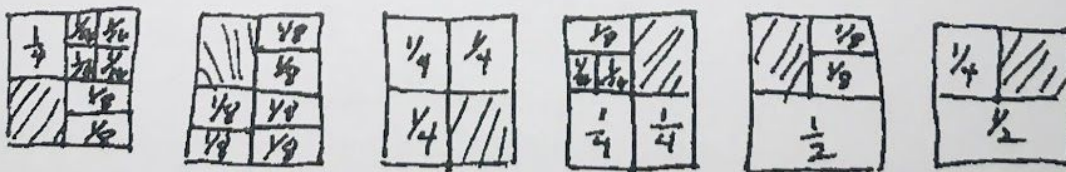


- b. It is interesting that until about grade 3 or 4, children often believe that one cake or the other will give a larger piece. If you have access to a young child, see what he or she says. (First make sure that the child agrees that the cakes are the same size.)
13. $\frac{5}{6}$ of an amount is represented here. How many asterisks would there be in $\frac{2}{3}$ of the same amount? Explain how you got your answer.



14. Children in one class were asked to show what $\frac{3}{4}$ means in as many ways as possible. Here is the work of three children. How would you evaluate the understanding in each case? (Make certain that you first understand what the child is thinking.)²

Sally:



Sam:

- a. $\frac{3}{4}$ is bigger than $\frac{5}{8}$.
 b. $\frac{3}{4}$ is smaller than 1 whole.
 c. $\frac{4}{4}$ is bigger than $\frac{3}{4}$.
 d. $\frac{13}{12}$ is bigger than $\frac{3}{4}$.

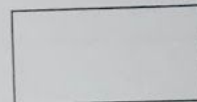
Sandy:

1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{16}$
-	-	-	-	12
-	-	-	1	10
-	-	-	2	8
-	-	-	3	6
-	-	-	4	4
-	-	-	5	2
-	-	-	6	-
-	-	1	-	8
-	-	1	1	6
-	-	1	2	4
-	-	1	3	2
-	-	1	4	-
-	-	2	-	4
-	-	2	1	2
-	-	2	2	-
-	-	3	-	-
-	1	1	-	-
-	1	-	2	-
-	1	-	1	2
-	1	-	-	4

Sandy claimed to have "found them all." What do you think he meant by that?
(Note that this is a table and that the first row contains headings for each column.)

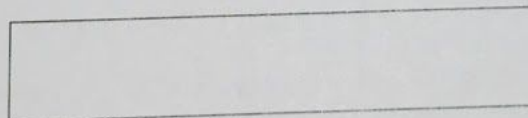
15. The given region shows $\frac{3}{4}$ of a candy bar.

- a. Show how large the original candy bar was. Write down your reasoning.
- b. Show how large $2\frac{1}{6}$ of the original candy bar would be.



16. The piece of carpet runner shown here is $2\frac{2}{3}$ units long (the units are usually yards or feet). Drawing as accurately as you can, show the units indicated in parts (a-c).
(Trace the original piece to make a separate drawing for each part.)

- a. $\frac{3}{4}$ unit
- b. 4 units
- c. $1\frac{1}{3}$ units



- d. If the $2\frac{2}{3}$ unit piece was for sale for \$4.80, what would be a fair price for 1 unit?
Write down your reasoning in terms of repeated unit fractions.

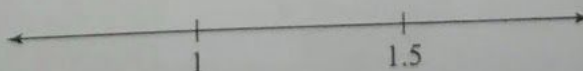
17. Pretend that the piece of expensive wire below is 1.25 decimeters long.

- a. Show how long 1 decimeter of wire would be (without using a ruler).



- b. If the 1.25-decimeter piece cost \$1.60, what would you expect the 1-decimeter piece to cost?

18. Here is part of a number line, with only two points labeled.



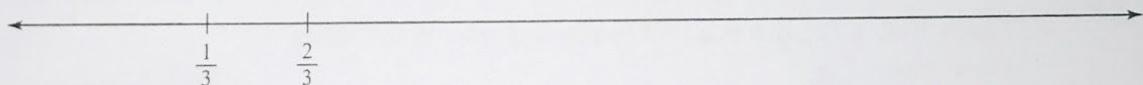
19. Use the part-whole notion to explain why $\frac{1}{7} > \frac{1}{8}$ and why $\frac{7}{10} > \frac{7}{11}$.
20. A second-grader announced to his father that $\frac{2}{7}$ was smaller than $\frac{1}{3}$ but bigger than $\frac{1}{4}$. When asked how he knew, the second-grader said, "You take 7 sevenths, and you know that 7 divided by 3 is bigger than 2, but 7 divided by 4 is smaller than 2." Can you explain his thinking?
21. Which label, discrete or continuous, applies to each of these "wholes"?

- the number of fans at a ball game
- the mileage traveled by a wagon train
- the amount of Kool-Aid drunk at a third-grader's party
- the number of cartons of milk ordered for snack time in kindergarten

22. Make a sketch to show each unit, and then use shading or some sort of marks to indicate the fractional part. Be sure that the thirds are clear.

- $\frac{2}{3}$ of the 6 children in the play were girls.
- $\frac{2}{3}$ of the 6 yards of cloth were needed for the play.
- $\frac{2}{3}$ of the 6 cartons of ice cream were chocolate.
- $\frac{2}{3}$ of each of the 6 cartons of ice cream were eaten.
- $\frac{2}{3}$ of the 6 moving vans were painted red.
- $\frac{2}{3}$ of each of the 6 moving vans were painted red.
- The store had 6 cartons of paper towels and used $\frac{2}{3}$ of them.
- The store had 6 cartons of paper goods and used $\frac{2}{3}$ of each of them.

23. Given a number line and only the points for the indicated fractions, accurately locate and label the points for the given numbers. If your markings do not make your reasoning clear, add an explanation. Extend the given line as needed.



- 0
- $1\frac{1}{4}$
- $2\frac{1}{6}$

24. Locate and label the points for the given numbers, as in Learning Exercise 23.



- 0
- $\frac{3}{4}$
- $\frac{7}{3}$

25. Mentally solve the equations in parts (a-c).

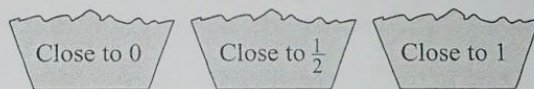
- $4 = \frac{x}{1}$
- $7 = \frac{y}{3}$
- $115 = \frac{z}{2}$

- d. Is it surprising that whole numbers can be regarded as naming fractions?

Learning Exercises for Section 6.2

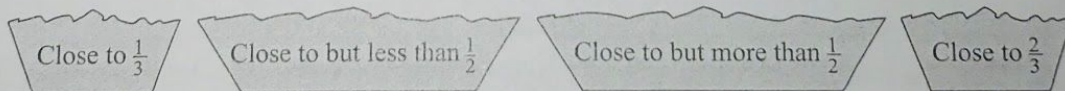
1. A sign says that all merchandise is being sold for a fraction of its original price. Your friend cynically says, "Yeah, $\frac{5}{4}$ of the original price." Why is this a cynical remark?
2. A student worked this problem: "The sewing pattern called for $\frac{7}{12}$ yard of silk and $\frac{5}{8}$ yard of linen. How much fabric was in the blouse?" He wrote $\frac{7}{12} + \frac{5}{8} = \frac{23}{24}$. Is the result of the calculation sensible? How can you tell by simple inspection, without actually performing the computation?
3. Is it possible for a fraction to have a value less than 0? Explain.
4. Draw lines from the fractions to the baskets in the figure below. Compare your answers with a classmate's when you have the opportunity. Discuss any differences. What did you do when a number could go in more than one basket?

$$\frac{4}{7} \quad \frac{2}{9} \quad \frac{11}{12} \quad \frac{99}{152} \quad \frac{17}{35} \quad \frac{15}{34} \quad \frac{11}{108} \quad \frac{3}{12} \quad \frac{9}{8}$$



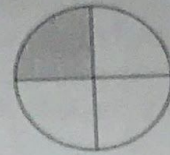
5. When is a fraction close to $\frac{1}{3}$? When is a fraction close to $\frac{2}{3}$? After you have answered these questions, draw lines from fractions to the appropriate baskets in the figure below.

$$\frac{4}{9} \quad \frac{21}{29} \quad \frac{49}{99} \quad \frac{15}{47} \quad \frac{27}{52} \quad \frac{42}{63} \quad \frac{31}{45} \quad \frac{22}{47} \quad \frac{99}{152} \quad \frac{2}{7}$$



6. In each case, tell which quantity is larger by thinking about how close or far each fraction is from 0, 1, or $\frac{1}{2}$.
 - a. $\frac{3}{4}$ or $\frac{4}{9}$ of the class
 - b. $\frac{1}{3}$ or $\frac{1}{4}$ of a cup
 - c. $\frac{3}{4}$ or $\frac{9}{10}$ of a meter
 - d. $\frac{2}{5}$ or $\frac{3}{6}$ of those voting
 - e. $\frac{13}{24}$ or $\frac{11}{20}$ of the money budgeted
 - f. $\frac{11}{25}$ or $\frac{17}{15}$ of the distance
7. How can you tell when a fraction has a value close to $\frac{1}{4}$?
8. Find a fraction *between* each pair of fractional values. Assume that the two fractions refer to the same unit. Justify your reasoning. (Do not change to decimal form.)
 - a. $\frac{2}{5}$ and $\frac{3}{5}$
 - b. $\frac{8}{15}$ and $\frac{9}{12}$
 - c. $\frac{7}{8}$ and $\frac{9}{10}$
 - d. $\frac{17}{18}$ and $\frac{17}{16}$
 - e. $\frac{1}{4}$ and $\frac{1}{3}$
9. A child claims that $\frac{4}{3}$ and $\frac{5}{6}$ are the same because "They are both missing one piece." What would you do to help this child come to understand these fractions better?

10. A child claims that $\frac{1}{4}$ can be greater than $\frac{1}{2}$ and draws the picture shown to support her argument. How would you respond to this child's claim? Can you adapt this picture to explain why $\frac{1}{2}$ is larger than $\frac{1}{4}$?



Cake



Cookie

11. Place the following fractions in order from least to greatest, giving careful explanations for how you knew their size relative to each other and benchmark numbers.

a. $\frac{4}{5}, \frac{4}{6}, \frac{5}{9}, \frac{2}{5}$

b. $\frac{11}{13}, \frac{7}{9}, \frac{22}{33}, \frac{5}{7}$

c. $\frac{31}{25}, \frac{27}{24}, 1\frac{1}{5}, \frac{23}{20}$

12. There are several strategies that can help you compare fractions. Make a list of strategies you have used or heard used in various activities or whole-class discussions. Give advice to a young child about when to use each of these strategies in ordering fractions. What would you suggest examining first?
13. Describe a mathematical practice in which you engaged in the activities and exercises in this class. Give an explanation with examples.
14. Estimate answers for each of the following fraction computations. Can you say whether your estimate is larger than or smaller than the exact answer would be? Remember what you learned earlier about number neighbors.

a. $\frac{5}{8} + \frac{9}{10}$

b. $1\frac{7}{16} + 4\frac{3}{27} + 7$

c. $14\frac{9}{10} - 5\frac{1}{16}$

d. $4\frac{11}{12} - 2\frac{15}{16}$

e. $\frac{20}{31} - \frac{1}{3}$

f. $4\frac{7}{16} + 5\frac{11}{12} - 2\frac{1}{8}$

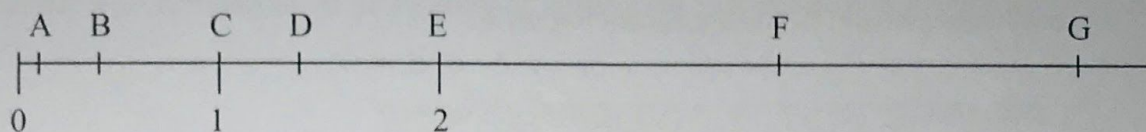
Learning Exercises for Section 6.3

1. a. Write 10 fractions that are equivalent to $\frac{2}{3}$ and 10 fractions that are equivalent to $\frac{5}{8}$.
 b. Do any of your 20 fractions have common denominators?
 c. When would having common denominators be useful?
2. Use sketches of regions or number lines to show each pair of equivalent fractions:
 a. $\frac{2}{3} = \frac{10}{15}$ b. $1\frac{4}{10} = 1\frac{2}{5}$ c. $\frac{5}{4} = \frac{15}{12}$
3. Find at least one equivalent fraction for each of the following fractions and demonstrate on a number line that the two fractions are equivalent:
 a. $\frac{3}{5}$ b. $\frac{3}{4}$ c. $\frac{0}{2}$
4. Suppose a school committee is made up of 6 girls and 4 boys. What fraction describes the part of the committee that is girls? How would you show, through a diagram, that $\frac{3}{5}$ also describes that part? (*Hint: GGGGGBBBBB can be organized as GG GG GG . . .*)
5. Write each fraction in lowest terms. For efficiency's sake, try to find the greatest common factor of the numerator and denominator.
 a. $\frac{450}{720}$ b. $\frac{24 \cdot 45 \cdot 17}{64 \cdot 12 \cdot 17}$ c. $\frac{225}{144}$ d. $\frac{x^2y^3z^6}{xy^4z^3}$ e. $\frac{(4.2 \times 10^7) \times (1.5 \times 10^8)}{4.9 \times 10^4}$
6. For the following pairs, place $<$, $>$, or $=$ between the two fractions. Use your knowledge of fraction size to complete this exercise.
 a. $\frac{3}{4}$ $\frac{3}{5}$ b. $\frac{102}{101}$ $\frac{75}{76}$ c. $\frac{8}{9}$ $\frac{40}{45}$ d. $\frac{8}{9}$ $\frac{9}{10}$ e. $\frac{13}{18}$ $\frac{29}{62}$
7. Using the context of a box of 24 crayons, illustrate that the fractions in each part are indeed equivalent.
 a. $\frac{4}{24} = \frac{1}{6}$ b. $\frac{18}{24} = \frac{3}{4}$ c. $\frac{4}{12} = \frac{2}{6}$
8. With the discrete model, choosing the unit to illustrate given equivalent fractions takes some thought. Just any number of objects may not fit the fractions well; for example, a set of 4 objects would not work for showing $\frac{2}{4} = \frac{3}{6}$. Describe a usable unit for showing that each of the following pairs of fractions is equivalent. Then show each fraction on a separate sketch of the unit, in such a way that the equivalence is visually clear. Make your sketch fit the context.
 a. $\frac{4}{5} = \frac{8}{10}$ Context: popsicle sticks
 b. $\frac{6}{9} = \frac{4}{6}$ Context: marbles (Do you have the smallest unit?)
 c. $\frac{2}{3} = \frac{4}{6}$ Context: children
 d. $\frac{2}{3} = \frac{12}{18}$ Context: Choose your own.
9. For illustrating that $\frac{3}{4} = \frac{9}{12}$, a student repeats a pattern that represents $\frac{3}{4}$ to show $\frac{9}{12}$, as in the following diagram. He explains that you can multiply the fraction by 3 to make an equivalent fraction. What problems are there with this diagram and his explanation?

$$X X X O = X X X O X X X O X X X O$$

What problems are there with this diagram and his explanation? What does a teacher will refer to *reducing* a fraction like $\frac{9}{12}$. What

11. Each letter on this number line represents a number. Match the numbers to each operation shown *without doing any calculation*. Use good number sense.



a. $\frac{3}{4} \times \frac{4}{3}$ b. $2 \times \frac{2}{3}$ c. $\frac{3}{4} \times \frac{1}{2}$ d. $2 \div \frac{2}{5}$ e. $\frac{1}{3} \times \frac{2}{5}$

12. In each case, use number sense rather than equivalent fractions to order these fractions and decimals. Use only the symbols $<$ and $=$. Explain your reasoning.

a. $\frac{2}{5}$ $\frac{2}{3}$ $\frac{40}{80}$ 0.60 $\frac{7}{10}$ b. $\frac{3}{15}$ $\frac{1}{10}$ 0.4 0.1 $\frac{1}{5}$ 0.2

13. Go to <http://illuminations.nctm.org/> and search for Equivalent Fractions in the Interactive Activities for Number and Operations. Do several sets of problems, as directed in the instructions, sometimes using circles, sometimes squares.

14. In the same activity given in Exercise 13, use the circle choice. Ignore the red circle; use only the blue and green circles to find which is the larger fraction. Use the $<$, $>$, and $=$ symbols as appropriate. You may need to use the number line below the circles.

a. $\frac{2}{3}$ $\frac{1}{2}$ b. $\frac{3}{5}$ $\frac{7}{11}$ c. $\frac{3}{10}$ $\frac{5}{12}$
d. $\frac{2}{5}$ $\frac{5}{16}$ e. $\frac{3}{4}$ $\frac{7}{11}$ f. $\frac{5}{14}$ $\frac{5}{16}$

Learning Exercises for Section 6.4

1. Determine which of the fractions in parts (a–f) have repeating decimals, and explain why. Then write each fraction as a terminating or repeating decimal.

a. $\frac{3}{8}$ b. $2\frac{3}{10}$ c. $\frac{3}{7}$ d. $\frac{3}{11}$ e. $\frac{9}{24}$ f. $\frac{5}{6}$ g. Two answers are the same. Why?

2. Write the following decimals as fractions, if possible. Change to simplest form or lowest terms.

a. 0.625 b. 0.49 c. 91.333... d. 1.7
 e. $0.\overline{9}$ f. $0.9\overline{3}$ g. $0.5\overline{3}$ h. $0.05\overline{3}$
 i. $4.\overline{76}$ j. $8.\overline{094}$ k. 0.1213141516171819110111112113114115...

[There is a pattern in part (k). Assume that the pattern you see continues. Is there a repeating block of digits?]

3. a. Are there any fractions between $\frac{7}{15}$ and $\frac{8}{15}$? If so, how many?

b. Are there any decimal numbers between $\frac{7}{15}$ and $\frac{8}{15}$?

4. Find decimal numbers between each pair of numbers [assume the pattern in part (d) continues]:

a. 0.4 and 0.5 b. 0.444 and 0.445 c. 1.3567 and 1.35677777...
 d. 0.00000111111... and 0.000001100110011...

5. Place the following in order from smallest to largest, without using a calculator:

$\frac{2}{3}$ $\frac{5}{6}$ $0.5\overline{6}$ 1.23 $\frac{3}{17}$ $\frac{12}{15}$ $\frac{11}{29}$ $\frac{11}{24}$ $0.2\overline{6}$ $\frac{1}{4}$ 0.21

6. Explain how you know there is an error, without calculating:

$$\frac{\text{some whole number}}{7} = 4.2923456789$$

7. One type of calculator can show only eight digits. Suppose the calculator answer to a calculation is 8.1249732. Explain how the exact answer could be the following:

a. a terminating decimal b. a repeating decimal c. an irrational number